Sovereign default and Output volatility

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Abstract

High volatility in the economic conditions inside a country seem to be related with sovereign defaults. This paper studies how changes in the output process can explain default events in emerging economies. It establishes that economic conditions inside the country, all else equal, have significant impact on the default decisions. Nevertheless, It may also be driven jointly with other factors such as political risk or small output costs.

Keywords: *Sovereign default, default probability, output process, output volatility.*

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1 Introduction

This paper studies the impact of output volatility and persistence of the output process in the default decision of governments in emerging economies. There are several causes that can explain why a country decides to default. First, output costs play an important role in the literature at explaining default decisions. When output costs are high, defaulting is costly for the government, and therefore we expect a low default probability. Conversely, when output costs are low we expect a higher default probability. Second, poor stability in the economic conditions may lead a country to enter into a debt default, so output volatility may also be important at explaining default decisions. Third, political shocks, such as political turnover may also explain default decisions. In this paper, I focus on the second cause. The main question of the paper is to which extent output process can explain the default decisions observed in the data.

A general fact that is in the data is the larger the volatility of output, the higher the default frequency (Figure 1). Economies with high volatility of output tend to have higher macroeconomic instability, which increases the probability of defaulting.

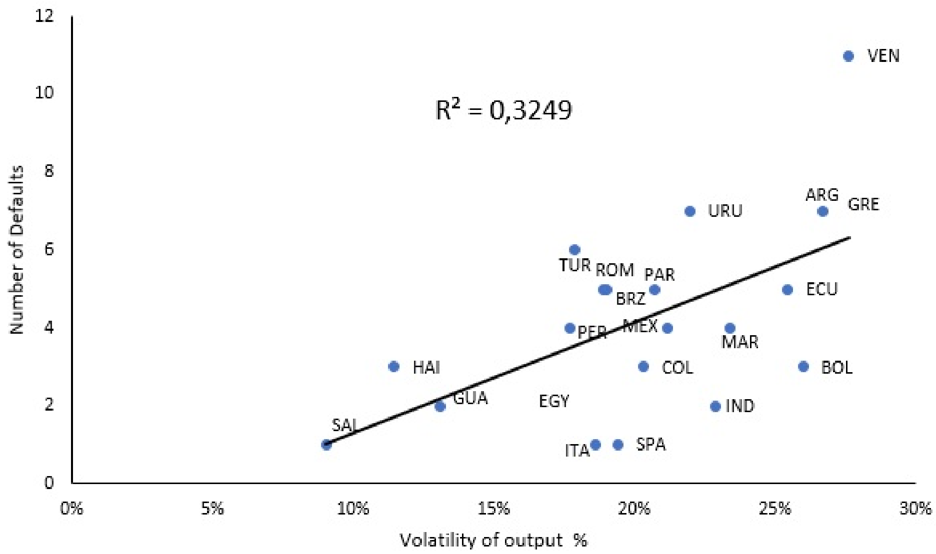


Figure 1: Volatility of output versus Number of defaults for twenty countries

In order to see whether the output process can explain default decisions in emerging economies, I replicate the stochastic general equilibrium model with endogenous default risk developed in Arellano (2008) for twenty countries. This is an extension of the approach developed by Eaton and Gersovitz (1981), where they study international lending, and how endogenous default probabilities and fluctuations in output are related.

In figure 2, I plot the persistence of the output process for each economy versus the volatility of output. Lower persistence is associated with higher volatility of output, which at the same time is related with a larger default frequency.

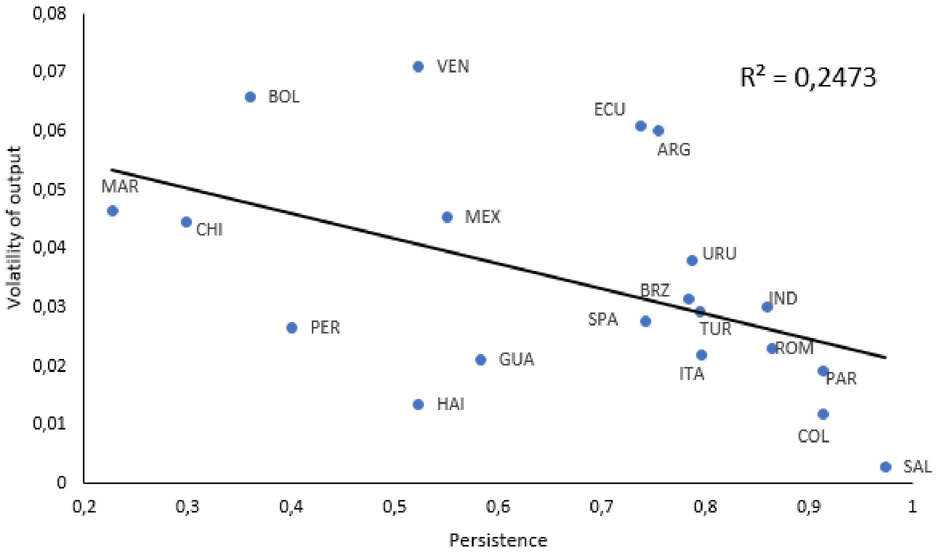


Figure 2: Volatility of output vs Persistence for twenty countries

As Panizza, Sturzenegger and Zettelmeyer (2009) point out, it is not clear what are the mechanisms that makes a country enter into default. It is easy to find a negative correlation between default and growth. However, it is much more difficult to test whether this negative correlation is driven by the default episode or by a series of other factors that are the cause of both the debt default and an economic recession. Authors in this literature have developed different models to explain default decisions in emerging economies. Hatchondo et al. (2009) shows that the presence of political turnover, which they call political risk, may affect borrowing and default decisions. In contrast, Arellano (2008), and Aguiar and Gopinath (2006) point out that output shocks are the main mechanism in the country's decision to default.

The focus of this paper is on understanding the interaction among the volatility of output and persistence with default decisions, in an environment of incomplete markets. My main contribution in the literature is on evaluating to which extent the output process can explain default decisions in emerging economies.

I use Colombian economy as a benchmark emerging economy. In table 1 I describe general statistics for this economy. Output and consumption are log and filtered with linear trend; the series start in 1970. Data is annual.

Table 1: Statistics of the business cycle in Colombia

Colombia	Mean	Std(x)	Corr(x,y)	Corr(x,r)
Output	24.39	1.37		-0.12
Consumption	25.29	0.67	0.82	-0.27
Trade balance	-0.006	0.028	-0.46	0.058
Interest rate spread	8.24	1.71	-0.12	

Third and fourth column shows correlations of each variable with output and interest rate spreads. Notice that output and consumption are negatively correlated with interest rate spreads. The trade balance is countercyclical and positively correlated with spreads, and interest rate tend to be high and volatile in these economies. These relations become much stronger in the default event because the output decrease sharply and interest rate increases dramatically in these cases.

2 Model

2.1 Set up

Consider a small open economy populated by a large number of identical households. Preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where x is the discount factor, c is consumption, and $u(.)$ is increasing and strictly concave. Each period, households receive y_t units of consumption goods. Households receive lump sum transfers from the government. The output shock is assumed to follow a Markov process, with transition function $f(y, y')$.

$$\ln\, y_t = \rho\, \ln\, (y_{t-1}) + \sigma_\epsilon \epsilon_t$$

The government issues one-period, non-state-contingent discount bonds, so markets of contingent claims are incomplete. The face value of these bonds determines the amount to be repaid next period, b_{t+1}. When the country purchases bonds $b_{t+1} > 0$, and when it borrows $b_{t+1} < 0$. The government cannot commit to repay its debt. As in the Eaton-Gersovitz model, when the country defaults it does not repay at date t and the punishment is exclusion from financial markets in the same period. The country reenters credit markets with an exogenous probability θ, and when it does it starts with a fresh record and zero debt.

If the government choose to repay the outstanding debt, the resource constraint of the economy will be given by,

$$c = y + b - q(b', y)b'$$

If the government defaults, the budget constraint reduces to $c = y^{def}$, where $y^{def} = h(y) \leq y$. With $h(.)$ being increasing in the income shock. We call this situation financial autarky.

The rest of the world is populated by a large number of risk neutral international investors and have access to international markets that can borrow or lend as much as needed at a constant international interest rate $r > 0$. Every period lenders choose bonds b' to maximize investor's profits π, taking price as given:

$$\pi = qb' - \frac{1-\delta}{1+r}b'$$

δ is the probability of default. Since firms make zero profits in equilibrium, the price of debt has to satisfy:

$$q = \frac{1-\delta}{1+r}$$

As in Arellano (2008), the probability of default δ is endogenous to the model and depends on the government incentives to repay its debt.

The timing of decisions is as follows: The government starts with initial assets b, observes the income shock y, and then decides whether to repay its debt or default. If the government decides to repay, then taking as given the bond price schedule $q(b', y)$, the government chooses b' subject to the resource constraint. After that, investors taking $q(b', y)$ as given choose b'. Finally, consumption c takes place.

2.2 Recursive equilibrium

In this section we define and characterize a dynamic recursive equilibrium. The government decides whether to default or repay its debt to maximize utility of the households. Define $V^0(b,y)$ as the value function for the government that has the option to default. Given this option, $V^0(b,y)$ satisfies

$$V^0(b,y) = \max_{c,d}\{V^r(b,y), V^d(y)\} \tag{1}$$

where $V^r(b,y)$ is the value associated with repaying the debt and $V^d(y)$ is the value of default. As in Arellano (2008), the value of default is given by

$$V^d(y) = u(y^{def}) + \beta \int_{y'} [\theta V^0(0,y') + (1-\theta)V^d(y')]f(y,y')dy' \tag{2}$$

When the government chooses to repay the debt, the value function becomes:

$$V^r(b,y) = \max_{b'}\{u(y - q(b',y)b' + b) + \beta \int_{y'} V^0(b',y')f(y,y')dy'\} \tag{3}$$

The government decision whether to repay or not its debt is a period-by-period plan. The country's default set can be characterized by $D(b) \subset Y$. It is the set of endowment shocks y's for which default is optimal given the debt position b.

$$D(b) = \{y \in Y : V^r(b,y) < V^d(y)\}$$

Denote the default policy by,

$$d(b,y) = \begin{cases} 0, & if \;\; V^r(b,y) \geq V^d(y); \\ 1, & otherwise \end{cases} \tag{4}$$

For this economy, a recursive equilibrium is defined as a set of policy functions for asset holdings $b'(b,y)$, default policy $d(b,y)$, consumption $c(b',y)$, and pricing function $q(b',y)$ such that:

1. Given the bond price function $q(b',y)$, the value function $V^0(b,y)$, the policies $b'(b,y)$, $c(b',y)$, and the default policy $d(b,y)$, satisfies the government optimization problem (1).

2. Bond prices satisfies the zero profit condition for investors:

$$q(b',y) = \int_{y'} \frac{1-d(b',y')}{1+r} f(y,y')dy' \tag{5}$$

3 Quantitative analysis

The second half of the 20th century is characterized by a large number of defaults from emerging economies. For the quantitative analysis I choose twenty emerging countries and calibrate Arellano's model to each economy. In particular, I identify the persistence of the income process and the volatility of output for the different countries.

The data is annual, and is taken from the World Bank. Output and consumption are log and filtered with linear trend; the series start in 1970. The trade balance data are reported as a percentage of output. The interest rate spread is the interest rate of each country minus the yield of the five-year US treasury bond. I take the default frequency from the database of Reinhart and Rogoff (2009) and it includes 120 years.

3.1 Calibration and functional form

The model is solved numerically to evaluate its quantitative predictions of debt decisions and default events in emerging economies.

I use a standard utility function $u(.)$ following:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The risk aversion parameter γ is set to two, which is common in the literature. The discount factor is calibrated to match the default probability in Colombian economy. The parameter related with the output cost, ϕ, is taken from Arellano (2008). To calibrate the probability of reentry I use the case of Colombian economy, which is a representative emerging economy. Cruces and Trebesch (2013) suggests an average exclusion period of 7.5 years for emerging economies after default events. However, Gelos et al. (2011) estimate the exclusion period as 4 years. A simple average of the above estimates gives 6.25. I round this number to 6.5 years. This estimate yields a value for θ of 0.154 at a yearly frequency. Which is the same as in Chatterjee and Eyigungor (2012).

As in Arellano (2008), I assume that default has a direct output cost of the form:

$$h(y) = \begin{cases} \hat{y} \;\; if \;\; y > \hat{y}; \\ y \;\; if \;\; y \leq \hat{y} \end{cases} \tag{6}$$

The asymmetric default output costs make the value of autarky a less sensitive function of the shock, which is important for giving the quantitative model the possibility to deliver the historical default probabilities as it is shown in Arellano (2008).

Table 2: Calibration of the model

Parameter	Value	Description	Author
γ	2	Risk aversion	Arellano (2008)
β	0.948	Discount factor	Calibrated
$r^\star$	1.7%	World interest rate	Arellano (2008)
θ	0.154	Probability of reentry	C & T., Gelos (2011)
ϕ	0.969	Output cost	Arellano (2008)
ρ_i	-	Persistence of $ln(y_t)$	Estimated
$\sigma_{\epsilon,i}$	-	Std dev. of ϵ_t	Estimated

The risk-free interest rate r is set to 1.7%, which is the average interest rate of a five-year US treasury bond during the period analyzed. Table 2 summarizes these parameters. They are common for all the countries, what is changing among the countries is the persistence and the output volatility of the process. For the case of Colombia, it is 0.913 and 0.0117, respectively. Table 8 in the appendix shows the persistence and output volatility of each country analyzed.

4 Results

After running Arellano's model using the algorithm described in the appendix, I study the behavior of the policy functions for the calibrated model and then evaluate its quantitative performance looking at the data. For Colombia I observe similar patterns as in Arellano (2008) in the bond price, equilibrium interest rate, value function, and savings, as it is shown in figure 3.

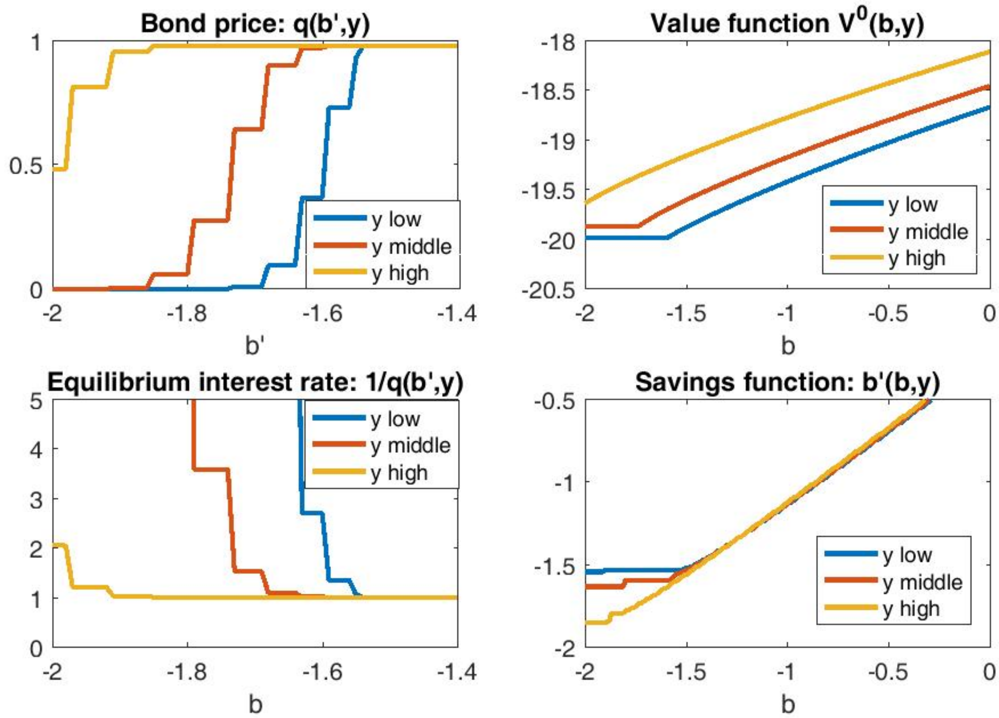


Figure 3: Colombia bond price, equilibrium interest rate, savings, and value function.

The borrower of the model has essentially two instruments to affect his time path of consumption: borrowing and default. The use of debt provides liquidity to the government, which in an environment of incomplete markets with income shocks can smooth consumption of the households across time.

Left upper panel in figure 3 shows the bond price as a function of assets b' for three different income shocks. Bond price is an increasing function of assets, which means that for high levels of debt, interest rate tend to be high, and as a result $q(b', y)$ should be small.

Notice that for periods of boom, i.e. high output, the probability of default is lower, and since the process is persistent, the price function for high income is to the left of the low output. The bottom left panel of the figure shows that when output is low the government chooses higher levels of debt and thus faces higher interest rates. The bottom right graph shows the savings policy function $b'(b, y)$ as a function of assets for different levels of output. When debt is large the economy save less as expected. As income decrease, the country would like to borrow more but it cannot because at these points it is usually at the constraint.

The second policy the government has is whether to default or not. The upper right panel of the figure 3 shows the value of the option to default or repay as a function of assets b for three different levels of output y. For a given output realization, default is chosen for all levels of assets below a threshold. The model predicts default for larger asset levels when output is lower.

4.1 Role of volatility of output in the default decision

In this section I evaluate the quantitative performance of the model when changing the volatility of output and persistence according to the output process of each country. In figure 4, I plot the average default probability observed in the data and the average default probability in the model. According with figure 4, all else equal, the output process is able to explain part of the default probability that we observe in the data. In particular, for countries with high volatility of output such as Venezuela and Argentina, the default probability delivered by the model is much higher than for countries with low volatility of output such as Egypt or El Salvador. The model is able to explain 75% of the variability in default events observed in the data. Furthermore, according to the model, once an emerging economy enter into default other emerging economies also tend to default, this is reflected by the positive correlation between data defaults and defaults predicted by the model.

(a) Default frequency (b) Debt to output ratio

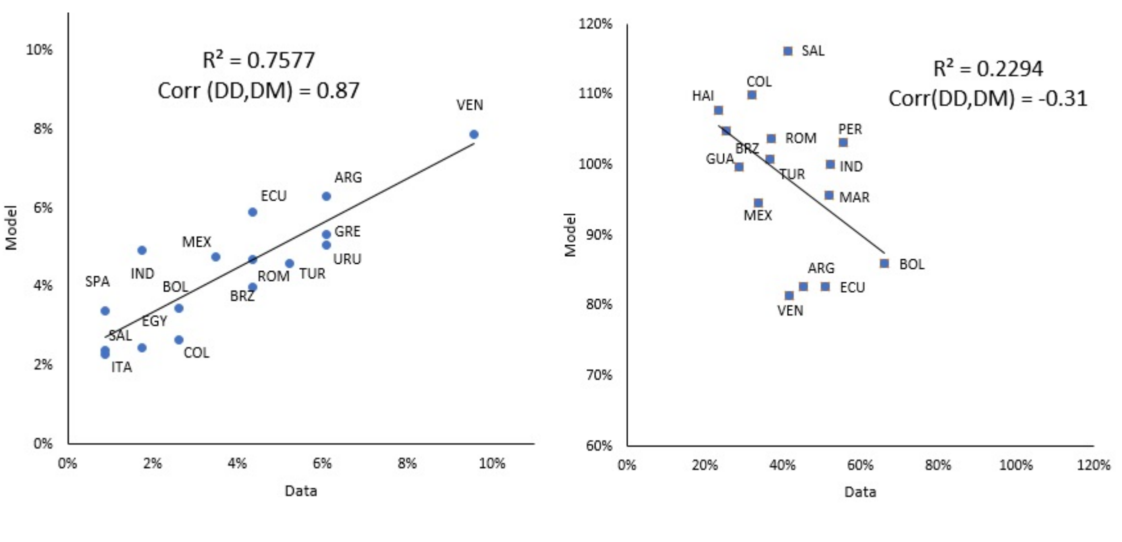


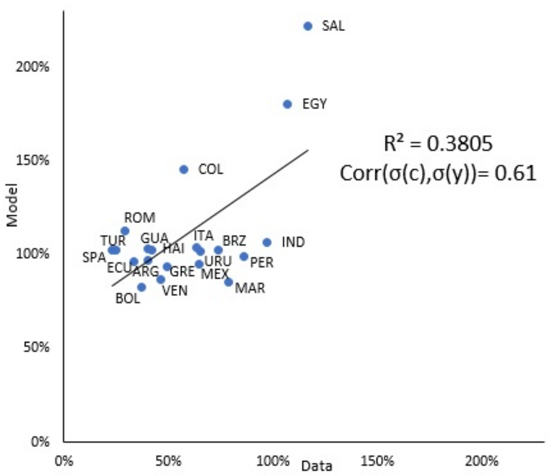
Figure 4: Data vs Model

Also notice that for some countries given the output process, the default probability delivered by the model is higher than the default probability observed in the data, while the opposite is true for other countries. This is due to the fact that I am not recalibrating the beta for each country, but I am assuming that all the countries have the same discount factor, which is not necessarily true, but for our purpose that is what we need. For a country with a default probability higher in the model than in the data, we need a higher discount factor in order to have a better approximation of the default probability given by the model. The opposite is true when the default probability delivered by the model is lower than the one observed in the data.

The right hand graph of figure 4 shows debt to output ratio in the data and the one delivered by the model. The model overpredicts debt to output ratio. However, countries with highest volatility of output have the lowest debt to output ratio, which is consistent with what we expect.

Finally, In figure 5 I plot the relative volatility of output and the correlation between consumption and output for the cross section. The left hand graph shows that the output process is able to explain almost 40% of the variability of the relative volatility of output that is observed in the data. Also, the correlation of the relative volatility among countries of output is positive, which represent the contagious effect of volatility of consumption in emerging economies that we generally observe in the data.

(a) Relative volatility: consumption and output



(b) Correlation: consumption and output

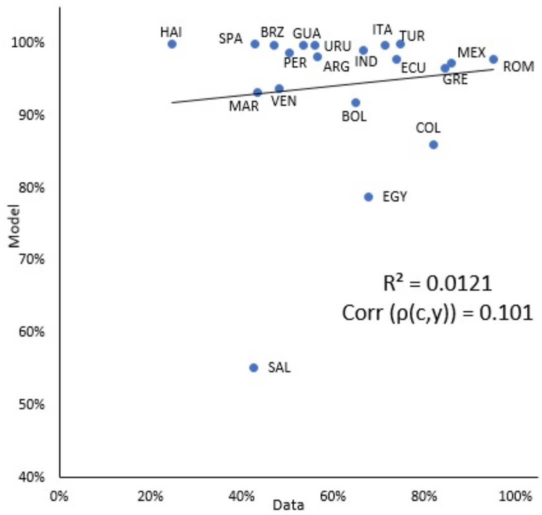


Figure 5: Data vs Model

On the other hand, the right hand side graph of figure 5 shows the correlation between consumption and output observed in the data and the one predicted by the model. The model is unable to explain this correlation observed in the data. However, for the case of Colombia, the model predicts the correct sign of this correlation.

According to these important results, economic conditions play an important role at explaining default decisions in emerging economies. On the other hand, debt to GDP ratio and consumption patterns are not very well explained by the output process of the economy. They may be also explained by other factors inside the country such as the behavior of interest rates or ”political shocks”. These results seem to contradict the results of Tomz and Wright (2007). They show that there exist a weak correlation between economic performance and default decisions, because countries have indeed defaulted during bad times, but they have also suspended payments when domestic economic conditions were highly favorable. However, in this paper what I show is that volatility of output and persistence of the output process can explain default events in emerging economies. In other words, volatile economic conditions help to explain sovereign default and not only bad or good economic conditions.

I do not discard that default events may also be explained by ”political shocks” as Tomz and Wright (2007) argues when they say that it may help explain the moderate correlation between output and default decisions found in the data.

In addition, default decision may have also been driven by the small output costs that emerging economies faced during part of the second half of the twenty century, where international banks lent huge amounts of money to Latin American countries, even when economic conditions where not stable, because they seem to have high growth levels in the following decades due to the boom in the price of commodities. The interaction between these possible causes of sovereign defaults in emerging economies are left for future research.

4.2 Simulation

The model is simulated for 300 periods for Colombian economy. I drop the first 100 observations, and plot some relevant statistics. To make the business cycle statistics comparable to the data, we choose the observations prior to the limiting distribution of assets as in the paper of Arellano (2008).

The model approximately matches the probability of defaulting, the countercyclical interest rate, and the countercyclical trade balance. Consumption in recessions is similar to output because borrowing is very expensive and the borrower is constrained. However, in periods of boom, debt is much cheaper, so it can be used to increase consumption. Table 3 present these statistics.

Table 3: Business cycle statistics for Colombia

	Data		Model	
	Corr(x, y)	Corr(x, r)	Corr(x, y)	Corr(x, r)
Output		-0.12		-0.009
Consumption	0.82	-0.27	0.8610	-0.011
Trade balance	-0.46	0.058	-0.22	0.0028
Interest rate spread	-0.12		-0.009	
Default probability	2.6%		2.65%	

The model matches the sign of the correlations at interesting; however, the main weakness of the model is that it underestimates the correlations between trade balance and output, and consumption and interest rate.

I also run simulations for many other countries of the sample and something interesting is that default rate delivered by the model is equal to zero for some countries in which Argentina is included. One may think that recalibrating the beta or changing other parameters may induce some positive probability of default; however this is not the case. For many other countries in the sample I have positive default rates.

5 Sensitivity analysis

In this section I study how sensitive the model is to the grid of debt and output, discount factor, persistence and output volatility of the output process.

5.1 Accuracy Tests

Since the model does not have closed form solution, its quantitative predictions are based on an approximation. Therefore, the validity of the model evaluation requires trust in the accuracy of the approximate solution. The question of how close the approximation is to the true solution cannot be answered with certainty because the latter is unknown. However, we can evaluate the approximation examining how stable the quantitative predictions of the model are to varying the number of grid points.

I want to evaluate whether the predictions of the model change as one increases the number of endowment points above the baseline value of n_y x n_b (21 x 201). Table 4 shows that this is not the case. The default frequency and debt policy are stable when I increase the grid of debt and output. Hence, I conclude that the baseline grid specification model yields a reasonable numerical approximation to the equilibrium policies of the Arellano's model studied here.

Table 4: Approximating Arellano's model for Colombia: Accuracy Tests

	Grid Points n_y n_b	Default frequency	$E(d/y)$
Data		2.6	32.13
Model*	21 201	2.65	109.94
Model	201 201	2.80	109.78
Model	401 201	2.78	109.82
Model	201 401	2.75	109.80
Model	401 401	2.75	109.80

5.2 The Role of Discounting

The predictions of the Arellano's model depends a lot on the assumed valued for the subjective discount factor β. One reason is that the lower is β, the higher is the household's desire for present consumption and therefore the higher the demand for debt.

The prediction of Arellano's model is also affected by β through the effects that it has on default costs. The reason is that default is penalized with financial autarky and an output loss for multiple periods. The more agents care about the future, the larger is the present value of these costs. Therefore, if β is increased we should expect a fall in the default frequency. Table 5 confirm the intuition given above. When β increases from 0.948 to 0.9698, default frequency decreases which implies that interest rate should decrease, and therefore debt to GDP ratio increases. This result suggest that the equilibrium level of debt is increasing in β.

Table 5: Sensitivity analysis: Role of discounting for Colombia

	Discount factor	Default frequency	$E(d/y)$
Data		2.6	32.13
Model	$\beta = 0.948$*	2.65	109.94
Model	$\beta = 0.958$	2.05	112.40
Model	$\beta = 0.968$	1.53	116.25

5.3 Changing the volatility of output process

An increase in the volatility of the output process for Argentina has two effects in the predictions of the model. One is that agents are more frequently exposed to positive or negative income shocks and as a result there is a higher incentive to default. The second effect is that an increase in uncertainty provokes a rise in precautionary savings and therefore a fall in the desired level of debt. These results are shown in table 6. Notice that increasing volatility of output from 0.0117 to 0.0217 increases the default frequency from 2.65 to 3.89, while the debt to GDP ratio decreases from 113.37 to 105.15.

Table 6: Sensitivity analysis: varying σ_e for Colombia

	Output volatility	Default frequency	$E(d/y)$
Data		2.6	32.13
Model	$\sigma_e = 0.0017$	2.09	113.37
Model*	$\sigma_e = 0.0117$	2.65	109.94
Model	$\sigma_e = 0.0217$	3.89	105.15

5.4 Varying the persistence of the output process

Finally, table 7 shows the predictions of the model for different values of ρ. As ρ increases the process becomes more persistent. For low values of ρ the default frequency is small. The reason is that when the endowment process is not highly serially correlated, the economy is expected to recover quickly when faces negative shocks. Therefore the probability of default is small. By contrast, when the endowment process is highly serially correlated, negative income shocks are expected to persist over time which makes the economy to default more frequently in bad states. Since the default frequency is low for low values of ρ, the economy is able to borrow more, resulting in high levels of debt. However, when ρ is very high, positive shocks are expected to last for a long period of time, making the economy to take on similar levels of debt as in the benchmark.

Table 7: Sensitivity analysis: varying ρ for Colombia

	Persistence	Default frequency	$E(d/y)$
Data		2.6	32.13
Model	$\rho = 0.5$	1.15	144.46
Model	$\rho = 0.75$	1.69	125.42
Model*	$\rho = 0.913$	2.65	109.94
Model	$\rho = 0.97$	4.48	118.36

6 Conclusions

This paper models endogenous default risk in a stochastic dynamic framework of small open economy with incomplete markets. The paper uses Arellano's model to study how changes in the output process affects the default decision for twenty emerging economies. It establishes that output process, i.e. persistence and output volatility, all else equal, is able to explain 70% of default decisions observed in the data.

The model matches the sign of the correlations for the business cycle statistics for Colombia. However quantitatively the model's correlations are lower.

The default and debt policy in the model are stable to changes in the grid points of debt and output, so the baseline grid specification model yields a reasonable numerical approximation of Arellano's model. The model is more sensitive to changes in the discount factor, output volatility and persistence. Default frequency is decreasing in β, and increasing in σ_e and persistence. On the other hand, debt policy is increasing in β, decreasing in σ_e and persistence. However, when persistence is very high, positive shocks are expected to last for a long period of time, making the economy to take on higher levels of debt as in the benchmark.

7 Appendix

Solution algorithm

Start by discretizing the debt and income space:

$$\mathbf{b} = b_1, b_2, ..., b_{N_b}$$

$$\mathbf{y} = y_1, y_2, ..., y_{N_y}$$

where $b_{N_b} = 0$.

Guess matrices representing the value functions: $\hat{V}_0(N_b \text{ x } N_y)(N_y \text{ x } 1)$, and $\hat{V}_0^d(N_y \text{ x } 1)$.

Guess a matrix representing the price function: $\hat{q}_0(N_b \text{ x } N_y)$

1. Using your guesses $\hat{V}_t$, $\hat{V}_t^d$ and $\hat{q}_t$, solve for every $i \in 1, ..., N_b$ and $j \in 1, ..., N_y$:

$$\hat{V}_{t+1}^c(i, j) = \max_{i' \in 1,...,N_b} u(y_j - \hat{q}_t(i', j)b_{i'} + b_i) + \beta \sum_{j'=1}^{N_y} \hat{V}_t(i', j')f(y_j, y_{j'})$$

$$\hat{V}_{t+1}^d(j) = u(h(y_j)) + \beta \sum_{j'=1}^{N_y} [\theta \hat{V}_t(N_b, j') + (1 - \theta)\hat{V}_t^d(j')]f(y_j, y_{j'})$$

$$V_{t+1}(i, j) = max\{\hat{V}_{t+1}^c(i, j), \hat{V}_{t+1}^d(j)\}$$

$\hat{d}_{t+1}$ denote the implied policy for default with dimension $(N_b \text{ x } N_y)$.

2.Using your policies from t+1, update the prices for every (i, j):

$$\hat{q}_{t+1}(i, j) = \sum_{j'=1}^{N_y} \frac{1 - \hat{d}_{t+1}(i', j')}{1 + r} f(y_j, y_{j'})$$

3. If $\|\hat{q}_{t+1} - \hat{q}_t\|, \|\hat{V}_{t+1} - \hat{V}_t\|$ and $\|\hat{V}_{t+1}^d - \hat{V}_t^d\|$ are small enough, you are done. If not, go back to step 1.

Arellano Code

Using the same parameters as in Arellano, I can replicate the graphs shown in her paper. In particular, I plot the bond price, equilibrium interest rate, value function and savings function for low and high income. This is shown in figure 7.

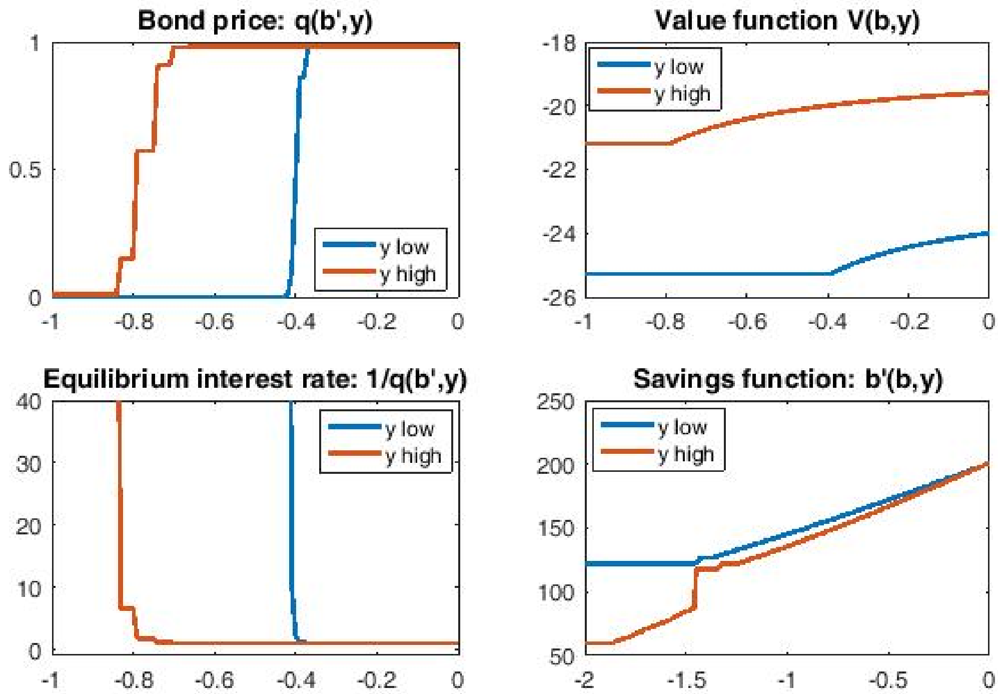


Figure 6: Bond price, equilibrium interest rate, value function, and savings function in Arellano (2008)

I use this code to study the relationship between default probability and output volatility for twenty different emerging economies.

Divide and conquer algorithm

We restrict the search from two sides using the algorithm developed by Gordon and Qiu (2015).

1. For any j, solve the problem at $i = 1$, searching everything.
2. Solve at $i = N_b$, setting $\underline{b}_t(i, j) = \hat{B}_t(1, j)$, $\bar{b}_t(i, j) = N_b$.
3. Set $\underline{i} = 1$ and $\bar{i} = N_b$.
4. Solve at $i = \frac{\underline{i}+\bar{i}}{2}$, setting

$$\underline{b}_t(i, j) = \hat{B}_t(\underline{i}, j), \bar{b}_t(i, j) = \hat{B}_t(\bar{i}, j)$$

5. Divide the grid in two at $\frac{\underline{i}+\bar{i}}{2}$. Conquer the two halves.
   (a) For the lower half, set $\underline{i}_{new} = \underline{i}$, $\bar{i}_{new} = \frac{\underline{i}+\bar{i}}{2}$, go to step 4.

(b) For upper half, set $\underline{i}_{new} = \frac{\underline{i}+\bar{i}}{2}$, $\bar{i}_{new} = \bar{i}$ go to step 4.

6. Continue until you solve for all i.

State space

The state space used in the code is shown in table 3.

Table 8: Discretization of State Space

n_y	21	Number of output grid points (equally spaced in logs)
n_d	201	Number of debt grid points (equally spaced)
$[\underline{b},\bar{b}]$	$[-2,0]$	Debt range.

Output processes

The output process for each country of the sample is shown in table 4.

Table 9: Output Process

Country	ρ_i	σ_i
ARG	$0,754$	$0,0602$
BOL	$0,36$	$0,0658$
BRZ	$0,784$	$0,0315$
COL	$0,913$	$0,0117$
ECU	$0,737$	$0,061$
EGY	$0,899$	$0,0074$
SAL	$0,974$	$0,0029$
GUA	$0,583$	$0,0211$
GRE	$0,501$	$0,0447$
HAI	$0,523$	$0,0134$
IND	$0,859$	$0,03$
ITA	$0,796$	$0,022$
MAR	$0,227$	$0,0465$
MEX	$0,55$	$0,0453$
PAR	$0,914$	$0,0193$
PER	$0,4008$	$0,0266$
ROM	$0,865$	$0,023$
TUR	$0,795$	$0,0292$
URU	$0,787$	$0,0379$
VEN	$0,523$	$0,071$

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